Volatility as an Asset Class: Holding VIX in a Portfolio

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Abstract

The ability to hedge market downturns without sacrificing upside returns has long been sought by all investors. We consider alternative methods of hedging the S&P 500 with assets that mimic the VIX index in hopes of taking advantage of the asymmetric relationship between volatility and returns. We first demonstrate that if the VIX was investable, and using the fact that volatility mean-reverts, can results in significantly improved portfolio performance over the buy-and-hold index portfolio. However, using VIX futures in a similar fashion does not provide the same results. As such, we deconstruct the VIX Index to find the relevant S&P 500 options that drive the VIX movements. In doing so, we then form a synthetic VIX portfolio using the S&P 500 options and capture returns similar to the VIX index. Our synthetic portfolio is highly liquid and investable, and when combined with a long position in the S&P 500, generates significantly higher returns with lower risk than the buy-and-hold S&P 500 index portfolio.

Introduction

Extreme stock market downturns are the most disconcerting periods for investors since they are risk averse, wish to limit volatility, and seek to avoid negatively skewed payoffs (Kumar (2009)). Millions of passive investors absorbed massive losses during the recent stock market retreat which reduced equity indices over 50% from October, 2007 to March, 2009. The anxiety of such movements even provoked many investors to sell large portions of equity, mutual fund, and exchange-traded fund (ETF) holdings, thus guaranteeing large losses.[1] The ability to more effectively hedge equity investments with holdings that would reduce portfolio downside risk over time could combat a considerable amount of investor unease.

We find that, due to the asymmetric and negative relation between VIX and S&P 500 returns, the VIX index provides a particularly effective hedge against market declines without proportionally penalizing performance when there are market gains. However, investors cannot directly invest in the VIX index, and only have exposure to VIX via the futures and options markets. Our results show that VIX futures are a poor hedging instrument, while VIX call options provide a good hedge with the added feature of capturing the positive skewness of the VIX index. We also demonstrate that a new portfolio strategy involving highly liquid S&P 500 call and put options can be constructed to mimic the VIX index and thus hedge market losses.

While a number of assets exist with returns that are, on average, negatively correlated with equities, these instruments may not provide the desired hedge against market downturns since the return correlations become positive in times of distress. For

[1] Mary Pilon, "Many Bought Shares High, Sold Low", *Wall Street Journal*, May 18th 2009.

example, during the market collapse in 2008, the value of both equities and commodities fell, though traditionally, commodities are negative beta assets.[2] Bond holdings also declined in value during the 2008 crash, as the need for commercial borrowing declined. Commercial and residential real estate values dropped greatly, in part due to excessive leverage, despite the fact that real estate is considered a counter-cyclical investment. Even many hedge funds, designed to cushion losses in equity markets, saw reversals over the 2007-2009 period. Szado (2009) documents the increased correlation of asset returns over the crisis period above the levels seen in the 2004-2006 period, implying that as the need for diversification grew, the ability of many assets to hedge equity holdings shrank at the most inopportune time. This is why we examine the important question of the ability of the VIX index to hedge downturns in the equity markets. Holding volatility as an asset class may make traditional negative or counter-cyclical investments moot for hedging purposes.

Since the introduction of VIX, it has widely been regarded as the economy's indicator of risk of the equities market. As noted by Whaley (1993, 2000), the VIX index is considered "the investors' fear gauge index." An important byproduct of introducing VIX was the newfound opportunity for investors to trade in futures, introduced in March 2004, and options, introduced in February 2006, thus allowing investors to enter into contracts and generate payoffs specifically related to volatility. While it is possible to invest in equity options and futures on the S&P 500 and construct a payoff that would be related to the volatility of the index, a more direct investment in VIX might require less management of the position and should provide no tracking error.

[2] Numerous commodity indices also retreated by more than 50% of value during the equity market decline of 2007-2009 as inflation ground to a standstill and consumption of raw materials slowed.

With more widespread access to market information, VIX has gained increased exposure in recent years, particularly as it rose to rare levels during the 2008 market decline. In the wake of increased attention, Whaley (2009) sought to clarify the meaning of VIX and discuss its characteristics. Whaley (2009) emphasizes that, like the S&P 500 index, the VIX index is not directly investable. However, while it is quite simple to replicate the payoff of the S&P 500 by holding the 500 underlying stocks in the appropriate proportions (or more simply, via investment in low-cost ETFs), it is difficult or impossible to replicate VIX by holding the underlying S&P 500 options. This is in part because VIX is constructed using the first two monthly expiration call and put out-of-the money options, with weights that are squared. Additionally, these weights change daily. Thus, even if a portfolio were able to hold the correct proportions on a given day, which would require a significant investment in many option contracts, the next day the proportions would change, and the rebalancing costs would be prohibitive. As such, the VIX index, viewed as an asset, has been practically untradeable.

With the availability of futures and options on VIX, it is possible to invest in volatility, or at least take a position on its future direction. However, it is unclear whether using the futures or options provides a payoff similar to that of the index, or provides the hedge against increases in volatilities that investor's most likely desire. As Szado (2009) notes, exposure to VIX calls and puts, as well as VIX futures, does not directly mimic holdings in the spot levels of VIX given that the mean-reverting nature of derivative instruments are priced into their values.

Because volatility mean-reverts, investing in the VIX index when it is low would provide protection against volatility increases. Dennis, Mayhew and Stivers (2006) and

DeLisle, Doran and Peterson (2010) document the asymmetric relationship between VIX and the S&P 500 and specifically show that VIX increases and S&P 500 declines are more strongly correlated than VIX decreases and S&P 500 increases. Whaley (2009) documents the mean-reverting nature of VIX and also describes its asymmetric nature such that VIX will rise more (less) dramatically during a stock market decline (rally). Furthermore, Simon (2003) notes the tendency of traders to overvalue (undervalue) the equity market when volatility levels are unusually low (high). Consistent with Daigler and Rossi (2006), it would appear that investing in VIX when it is low not only provides a hedge against declines in the S&P 500, but will not proportionally penalize investors when the S&P 500 increases.

A number of authors have thus considered the possibility of hedging portfolios with VIX-mimicking assets. Dash and Moran (2005) initially considered the ability of newly formed VIX-based products to lower portfolio risk. Emerging possibilities then developed for such a strategy, including the use of VIX futures, VIX options and VIX-based ETFs. Brenner, Ou and Zhang (2006) introduce an option on a straddle designed to hedge volatility risk. This instrument is sensitive to volatility innovations and thus useful as a hedge. Windcliff, Forsyth and Vetzal (2006) consider the variations in the contract designs of volatility derivatives and discuss the difficulties of hedging the returns with such instruments, particularly given delta and delta-gamma hedging techniques. Black (2006) and Moran and Dash (2007) find that adding VIX futures to a passive portfolio can significantly reduce portfolio volatility. VIX's quick movements during risky markets also improve the skewness and kurtosis of the overall portfolios. Briere, Burgues and Signora (2010) advocate a sliding approach when hedging in which more (fewer) VIX futures contracts are held when VIX levels are notably lower (higher) due to the mean-reverting nature of the index.

An important remaining question is the cost-effectiveness of using VIX as a hedging strategy. If VIX were tradable, we hypothesize that investing in the S&P 500 and VIX, when VIX is relatively low, would provide investors with a portfolio that will increase in value when the S&P 500 increases and will be somewhat protected when the S&P 500 falls. Thus, we wish to explore the profitability of VIX-hedged portfolios to assess the implications of holding the VIX index alongside the S&P 500. In part, this should reveal whether investing in the VIX provides an effective hedge to long-equity positions, either through lower costs or superior returns, than alternative hedges, such as purchasing index puts.

We then explore the benefits of investing in VIX futures and study whether the futures contracts mimic the payoff to the VIX index. Since these are volatility products that are tradable, if the payoffs do not replicate the underlying index, it may imply that a tradable asset on volatility does not provide investors with appropriate protection. Our results show that this is the case for VIX futures. Next, we examine the benefit of using VIX call options for portfolio insurance, relative to S&P 500 put options, to assess whether the payoffs to VIX options provide similar downside protection. We find that VIX options provide a reasonable hedge, as well as capture the positive skewness of VIX. However, VIX options are thinly traded in comparison to S&P 500 options, and may present an illiquidity problem. Finally, we construct a low-cost portfolio that attempts to capture the increases in VIX by exploring which S&P 500 options drive the changes in VIX. We demonstrate that, by deconstructing VIX into the individual option components, it is possible to form a portfolio of liquid S&P 500 options that captures the payoff to the VIX index which eludes the future and option contracts on VIX. This synthetic VIX position improves the hedging prospects of otherwise passive, long-equity investors.

The rest of the article is as follows: Section I presents the data and its sources, Section II presents the analyses of the performance of VIX as a hedge against market declines, Section III investigates which parts of VIX make it a favorable hedge and evaluates a low-cost portfolio constructed to mimic VIX returns, Section IV concludes the paper.

I. Data

The data for comparison of VIX-based hedging strategies is assembled from numerous sources. The actual daily VIX levels, utilized for construction of the theoretically ideal strategy are taken from the CBOE's historical website.[3] The methodology for VIX construction was amended in September, 2003, though retroactive calculation allows for collection of data beginning in 1990. Historical S&P 500 returns are taken from CRSP in order to calculate the returns of positions that theoretically hedge S&P 500 holdings with the raw VIX level. To adopt actual S&P 500 holdings, we gather price data for the SPY ETF, which mimics the performance of the S&P 500, also from CRSP.

Futures positions are used as one method for creating a VIX-hedged portfolio. VIX futures began trading on the CBOE futures exchange in March of 2004. We collect the daily prices of VIX and S&P 500 futures contracts from the *Wall Street Journal* archives, beginning with the arrival of VIX futures.

A second method for assembling a tradable VIX-hedged portfolio is based on the use of VIX options. These options began trading on the CBOE in February of 2006, and we

[3] http://www.cboe.com/micro/VIX/historical.aspx

subsequently collect their daily prices from Optionmetrics in order to track the performance of long-S&P 500 portfolios which are hedged with VIX options.

Additionally, we collect daily data for the S&P 500 index options, SPX, beginning with the initialization of Optionmetrics in 1996. SPX calls and puts are used to create our synthetic VIX position, whose returns are tracked through time. While we focus on the hedging ability that VIX futures and options as well as our synthetic position provide for holdings of the ETF SPY, we alternatively consider the hedging abilities of S&P 500 put positions.

Table 1 presents the summary statistics for S&P 500 and VIX returns. Panel A shows that S&P 500 returns over the entire sample period average 50 basis points per month, while VIX returns average 164 basis points per month. However, the standard deviation of the VIX monthly returns is 17.79%, which is so volatile that the average VIX returns are statistically indistinguishable from zero. The correlation between S&P 500 and VIX returns is -0.616, demonstrating a strong negative relation. Panel B limits the sample to months in which S&P 500 returns are positive. Limiting the sample in this manner yields average S&P 500 and VIX returns of 3.34 and -5.96 percent per month, respectively. The correlation between S&P 500 and VIX returns during these months is -0.274. Panel C shows that the average S&P 500 and VIX returns are -3.51 and 12.65 percent per month, respectively, when the sample is limited to months where S&P 500 returns are negative. The return correlation for these months is -0.574. When separating the returns into up and down periods, mean returns for both the S&P 500 and VIX are significantly different from zero, highlighting the importance of looking at different market conditions. More interestingly, the absolute value of the mean S&P 500 returns is similar for up and down market conditions, while the positive mean VIX returns are twice as large during periods of negative S&P 500 movement as the negative mean VIX returns in periods of positive S&P 500 returns. This result highlights the asymmetric relation between S&P 500 returns and VIX returns and may be indicative of the potential for using volatility as a hedging instrument against S&P 500 losses.

II. Effectiveness of VIX as a Hedge Against Market Losses

A. VIX Index as a Hedge

We next examine whether forming portfolios using the VIX index can result in higher or less volatile returns than passive investing in the S&P 500. While it is not possible to buy the VIX index, we nonetheless wish to examine whether theoretical holdings in VIX benefit investors by capturing both the asymmetric relationship between VIX and the S&P 500 and the mean reversion in volatility. We initially examine investing equally in the S&P 500 and the VIX index, and then, given the mean-reversion properties of VIX, we consider investing in the VIX index only when its level is below certain thresholds. In doing so, the intent is to capture periods when VIX increases while only investing in the S&P 500 fully when VIX is falling. The results from the portfolio analyses, based on buying various combinations of the S&P 500 and VIX indices, are shown in Table 2.

A portfolio made up of strictly the VIX index, shown in Panel A, has a monthly mean return over the entire sample that is not significantly different from zero, but has statistically significant CAPM and Carhart (1997) 4-factor model (henceforth just "4-factor") monthly alphas of 2.15 and 2.29 percent, respectively. The estimated market betas in the CAPM and 4-factor models are both -2.13 and are highly significant, which is consistent with the negative relationship between the VIX index and the S&P 500. In months with positive S&P 500 returns, the alphas are negative and highly significant while the CAPM market beta and 4-factor market beta are statistically insignificant. Thus, there appears to be only a weak relation between VIX and market returns when S&P 500 returns are positive. When S&P 500 returns are negative, however, the relationship between VIX returns and the S&P 500 is statistically significant with a CAPM market beta of -1.70 and a 4-factor market beta of -1.60. If the VIX index could be bought, this asymmetric relationship would imply that the VIX index can hedge decreases in the S&P 500 without penalizing the portfolio when the S&P 500 increases. However, even though the relationship is insignificant, the 4-factor alpha accompanying an S&P 500 increase is -5.07%, and thus it may be desirable to avoid VIX holdings when the S&P 500 is increasing.

The pertinent question, of course, is when to invest in VIX and when to concentrate holdings in the S&P 500. While it is practically impossible to forecast when the market is going up or down, forecasting VIX movements may be easier due to the historically mean-reverting nature of volatility. Ahoniemi (2008) demonstrates that an ARIMA (1,1,1) model significantly predicts the directional changes of VIX, thus indicating the mean-reverting tendency of the index. The historical mean for the VIX index is 22%, which corresponds to the historical average of volatility for market returns of 19.8%. The VIX index's mean-reverting property will tend to move the index back towards the mean when it deviates greatly from that mean, while the S&P 500 will typically respond in the opposite direction. Therefore, a naïve strategy of buying the VIX index when it sinks below a certain level may hedge against future decreases in the S&P 500. To investigate the benefits of investing in VIX, we consider multiple upper bounds of VIX for such naïve portfolio construction.

Panel B of Table 2 presents the performance of portfolios consisting of 50% in the S&P 500 and 50% in the VIX index when VIX is below an indicated level and entirely in the S&P 500 otherwise. The portfolio is rebalanced at the end of each month. A Naïve 20 strategy sets the maximum VIX-buying level at 20, meaning the portfolio buys VIX if the end-of-the-month VIX is below 20 and will sell any VIX holdings and place all funds into the S&P 500 if VIX rises above 20. The mean return for this portfolio is statistically significant, earning 1.54% per month while the CAPM (4-factor) alpha is 1.00% (1.25%). The portfolio's market beta for the CAPM (4-factor) case is now a positive 0.48 (0.41), up from the -2.13 reported in panel A and highlighting the effect of VIX on portfolio returns by capturing positive returns in VIX when the S&P 500 falls. Thus, the Naïve 20 portfolio is positively related to market returns, but the systematic risk of the portfolio is half that of the market portfolio and the portfolio returns outperform market returns, even when size, book-to-market equity ratio, and momentum are accounted for by the 4-factor model.

The allowable VIX-buying level is increased to 25 in the Naïve 25 strategy. The performance of this portfolio is similar to that of the Naïve 20 portfolio. The main difference between the two portfolios is that the Naïve 25 portfolio has a market beta that is not statistically different from zero, indicating that the portfolio has no exposure to systematic risk. As the VIX-buying level is increased from 25, the portfolios mean returns and alphas remain similar. However, as the threshold level increases, the market beta of the portfolio becomes more negative and increasingly significant. For example, the CAPM market beta for the Naïve 30 portfolio is a statistically significant -0.23 and decreases to -0.43 for the Naïve 40 portfolio. Increasing the threshold level increases the negative exposure to systematic risk by holding the VIX index in the portfolio more often.

Figure 1 displays the market values of the S&P 500 portfolio and the portfolios based on the Naïve 20 and Naïve 40 strategies. Each portfolio starts with $100,000 at the beginning of the year in 1990. Although all naïve portfolios outperform the S&P 500 portfolio, which has a final portfolio value of $0.18 million, and demonstrate the hedging ability of the VIX index, the buy-and-hold returns of the strategies differ dramatically. The Naïve 20 portfolio has an ending market value of $2.12 million, while the Naïve 40 portfolio has an ending value of $0.92 million. Thus, while the mean returns and alphas appear similar, the buy-and-hold (BHR) returns tell a very different story. Table 3 shows the BHRs for each portfolio and the corresponding risk.

B. VIX Futures as a Hedge

The portfolios examined above rely on the ability to buy and sell the VIX index which, currently, is not a possibility. However, since April 2004, it has been possible for a portfolio to buy or sell cash-settled VIX futures. Since entering into VIX futures may be a close substitute to theoretical investment in the VIX index, we wish to investigate whether investing in the VIX futures actually replicates the payoff of the index. Panel A of Table 4 shows the mean returns and alphas from a portfolio that invests in one-month VIX futures contracts, entering into new contracts when the prior month contracts expire.

A portfolio that invests in VIX futures, starting with the availability of futures data in April 2004, through the end of August 2009, returns a statistically insignificant -3.12% a month. By comparison, the monthly return of the VIX index over the same period was 1.8%

per month. When separated into periods of positive and negative S&P 500 returns,[4] the returns to VIX futures contracts are more negative when the S&P 500 is increasing, while the opposite is true of the VIX index itself.[5] The difference between mean returns of the VIX futures and the index were -4.73% when S&P 500 futures fell and -6.84% when S&P 500 futures rose. Thus, the holders of futures contracts overpaid relative to the index regardless of the direction of the S&P 500 futures returns. This contrast between the VIX futures and the index suggests the futures contracts did not offer the same downside protection investors would expect given the asymmetric relationship between volatility and returns.

Panel B of Table 4 demonstrates the poor performance of the naïve strategies using futures contracts on the S&P 500 and VIX indices.[6] The mean returns and alphas of all the portfolios are negative and statistically significant for the Naïve 20 and Naïve 25 futures portfolios. Figure 2 displays the market value of the Naïve 20 and Naïve 40 futures portfolios over time, along with a portfolio in S&P 500 futures, and demonstrates the poor performance of incorporating VIX futures in a portfolio. By comparison, over the same holding period, the Naïve 20 portfolio using the actual VIX index would have doubled in value. The poor performance of the futures may be due to their term structure where sellers of the futures incorporate a premium for the upside risk in the index futures, since on average, VIX futures have an upward sloping term-structure.

[4] In order to match the term structure of the VIX index futures, the portfolio should contain S&P 500 index futures of similar maturity rather than the S&P 500 index itself.
[5] For the same periods, when the S&P 500 was falling (rising), the VIX index increased (decreased) on average by 17.0% (-8.2%).
[6] Results are similar using the S&P 500 index returns.

C. VIX Options as a Hedge

Since VIX calls became available to trade in March 2006, we compare the hedging ability of these options to using S&P 500 puts to hedge downside risk exposure to market movements. Specifically, we are interested in two aspects of VIX calls as an alternative type of portfolio insurance; first, the cost of the insurance, and second, the options' ability to capture the asymmetric relation between volatility and returns.

The construction of the hedged portfolio assumes a long position in the S&P 500 index by purchasing shares of the index-tracking ETF, SPY. The portfolio holds the initial position, or shares in the SPY, constant throughout the evaluation period. In order to hedge this position, the portfolio simultaneously purchases the requisite number of near-term, ATM, S&P 500 puts or VIX-Index calls, and holds the contracts to expiration. On the following Monday, the portfolio then enters into new near-term contracts, hedging the same number of underlying shares. This keeps the hedge position identical each period as only the cost of the hedge, which is a function of market volatility, and the payoff to the options, will vary. This allows us to keep track of the cost, payoff, and effectiveness of each strategy by observing the change in portfolio value. The hedge portfolio will alternatively utilize 5% OTM contracts for both S&P 500 puts and VIX-Index calls to account for potential moneyness effects.

Since we wish to compare the cost and performance of the two hedging strategies, we assume the same dollar value of options is purchased each month for both S&P 500 puts and VIX Index calls. As a result, many more VIX options may be purchased. For example, if the S&P 500 index level was 1067, and the portfolio held 100 shares in the index, we could buy one put with a strike price of 1065 for $30. If the VIX index was at 25, the alterative portfolio could buy 10 calls at a price of $3 each.[7] Each position would cost $3000. If the index fell to 1000, the put would result in a payoff of $6500 minus the original $3000 cost. The payoff to the VIX calls would be equivalent to the S&P 500 put if the index increased to 31.5. Ultimately, we are interested in discovering how the VIX index tends to react in these situations, and whether it is a more or less effective hedge when the S&P 500 is falling. Alternatively, the portfolio could hold fewer VIX-call options, though it is easier to assess the performance of the hedge by allowing only the payoffs from the options to deviate.

Figure 3 demonstrates the performance of the VIX-call and S&P 500-put hedge strategies from March 2006 through October 2009. From the beginning of the evaluation period through the summer of 2008, there are instances when the payoff to one strategy may be slightly higher than the other, but the value of the portfolios remain similar. This is unsurprising given the high negative correlation between the VIX index and the S&P 500. It is also not surprising that the hedge portfolios underperform while the S&P 500 was generally growing steadily since the cost of portfolio insurance is expensive (Bates (2000)). However, starting in September 2008, when the markets began falling and volatility rose to historic levels, the payoff of the VIX-call strategy drastically outperforms the S&P-put hedge. While this is partly the result of holding more call options, it is mainly due to the ability of VIX calls to capture the positive skewness in volatility that the S&P 500 puts cannot capture. The fall in the S&P 500 from July through November 2008 was over 36%, but the corresponding increase in the VIX index was 300%. This resulted in an increase in value of about 50% in the portfolio that used VIX calls as a hedge, while the portfolio that used S&P 500 puts still declined 20%. This suggests that to hedge significant market

[7] Actual prices on 2/11/2010

declines, calls on volatility are better instruments to use than puts on the S&P 500 index because positive skewness in volatility is unbounded.

Hedging simple, passive positions like long-S&P 500 with VIX calls, rather than S&P 500 puts, appears to be an attractive alternative even though both options are expensive. It is the ability of VIX calls to capture positive skewness that suggests they are a superior alternative. What remains unclear is whether the difference in the payoffs in the two hedging strategies is due to underpricing of VIX calls or overpricing of S&P 500 puts. There is recent evidence that suggests negative skewness is priced in equity returns (Boyer, Mitton and Vorkink (2009)) and index option prices (Doran and Krieger (2010)). Our portfolio result suggests that the options on volatility have not incorporated corresponding positive skewness or the increase in skewness in VIX relative to the decrease in skewness in the index. As such, it appears that VIX calls are cheap only because of the differences in underlying return distributions of volatility and equity instruments, not necessarily because they are mispriced.

III. Replicating the VIX index

A. Decomposing VIX

While numerous studies have analyzed the impact of the VIX Index, or changes in VIX, on the returns of equity portfolios, few have considered what drives the actual changes in VIX itself. Whaley (2009) examined the effect of changes in the S&P 500 index on VIX changes and found an asymmetric response to those changes. This is not surprising and consistent with the results of DeLisle, Doran and Peterson (2010). However, to this point, there has been no examination based on the actual options that cause VIX to change. Since the VIX Index is constructed using only near-term and 2nd near-term month closest to ATM and all out-of-the money puts and calls on the S&P 500, it is worth testing which options drive the movement of VIX. Since there is an asymmetric response between changes in the S&P 500 and VIX, it is highly likely that puts versus calls, ATM versus OTM, or near-term versus 2nd near-term monthly expiration discrepancies are responsible for the changes.

Before deriving an empirical specification to test the relationship between S&P 500 options and VIX changes, we briefly summarize the methodology used to calculate the VIX Index. The specification for VIX is given by:[8]

$$\sigma_j^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F_j}{K_0} - 1\right]^2 \tag{1}$$

$$VIX = 100\sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30}-N_{T_1}}{N_{T_2}-N_{T_1}}\right]\right\}\frac{N_{365}}{N_{30}}} \tag{2}$$

where K is the strike price of options that are currently closest to ATM or OTM, ΔK is the difference in strike prices, $Q(K)$is the midpoint price of the option at strike K, $T_1$is the expiration of the near-term option, $T_2$is the expiration of the 2nd near-term option, N is the minutes to expiration and F represents the ATM forward price of the index. Using all options available that are closest to ATM and OTM, equation (1) constructs a σ_j^2 for both the near-term and 2nd near-term options, while equation (2) weights the σ_j^2 to create a 30-day measure of implied volatility.

While both equations are straightforward to calculate and recreate the current level of VIX, the non-linear nature makes replicating the payoff to the VIX index difficult to

[8] A *white paper* available from CBOE at http://www.cboe.com/micro/vix/vixwhite.pdf shows the construction of the VIX index.

accomplish. The weight on each option is a function of a root-weighted time variable, which changes daily, making rebalancing costs prohibitive while creating indivisible option units. However, it is possible to unwind the formula into smaller components, allowing for a test of the effect of each option, or the changes in the prices and moneyness of each option, on the changes in VIX.

Taking the natural log of equation (2) removes the nonlinear term such that equation (2) become linear, and allows for a simple regression to assess the impact of the individual options on the change in VIX. Since we are interested in changes, testing the difference between $\ln(VIX_t) - \ln(VIX_{t-1})$, is equivalent to testing $\ln\left(\frac{VIX_t}{VIX_{t-1}}\right)$, or the percentage change in VIX each day. The key explanatory variables to include are whether the option was a call or a put, the time to expiration of the option, the price change, the strike price or moneyness change, and an interaction term between these variables. It is necessary to incorporate an interaction term, since the first term in equation (1) is multiplicative in these variables.

We run the following specifications, controlling for time variation,

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t}$$
$$+\Delta P_{\kappa,\phi,\tau,t} * CD_t + \Delta KS_{\kappa,\phi,\tau,t} * CD_t + \varepsilon_t \quad (3)$$

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \varepsilon_t \quad (4)$$

Where $\Delta P_{\kappa,\phi,\tau,t}$is the change in price of ϕ(call/put) option with a strike of K and maturity τ at time t. ΔKS represents the change is moneyness of the option, where put moneyness is equal to S/K and call moneyeness is equal to K/S. Thus, more positive changes bring both definitions of moneyness closer to ATM. DM is the time to maturity of the option, and I is the interaction of all three variables. CD is a dummy variable equal to one if the option is a call and zero if the option is a put. Equation (3) interacts the dummy variables with change in price and moneyness variables to capture differences between call and put options. Equation (4) examines the effect of price and moneyness changes of calls and puts separately on VIX changes. Each specification clusters on the date to avoid overstating the t-stats. The results are shown in Table 5.

We conduct seven estimations with the first three using equation (3) and the final four using equation (4). The first utilizes the full sample while the second (third) considers the positive (negative) VIX changes only. The fourth and fifth estimations use only puts and segment the sample based on whether VIX is above and below the historical mean of 22. The last two estimations use only calls, again segmenting the sample based on VIX levels above and below 22.

The results of the first estimation reveal that increases in price are positively related to VIX changes. Specifically, if the option becomes one dollar more expensive, then VIX will increase by 0.25% on average. However, if the option is a call, for each dollar increase the change in VIX is only 0.11% as the coefficient of the call dummy is -0.14. This suggests a greater impact from put price changes than call price changes. Options that are closer to ATM have a greater effect on VIX values, as a 0.01 increase in moneyness results in an increase of 3.32% in VIX. Again, this is only for puts, because as call options approach ATM status, the effect on VIX is a fall of -4.01%. This is consistent with negative correlation between VIX and the S&P 500 since puts (calls) become more expensive as the S&P 500 falls (increases). There is no significant relationship with the expiration of the option, although the negative coefficient on DM is consistent with a negative option theta. The interaction term is positive, as expected, but the effect on the change in VIX is small.[9]

The results of the second and third estimation are similar in direction for all the coefficients, but the interpretation of the coefficients reveals interesting results. The coefficient on ΔP is 0.141 larger and statistically different than the ΔP coefficient when VIX is increasing versus decreasing, suggesting an asymmetric response to put price changes on VIX changes. Also the discrepancy between put and call price changes on VIX changes is larger when VIX is increasing. When VIX decreases, $\Delta P * CD$ is insignificant, again highlighting the importance of put price changes for VIX increases, which are highly negatively correlated with negative S&P returns. The results of the final four estimations further emphasize these findings. Put price changes have a greater effect on VIX changes at all levels of VIX, relative to calls, but the effect is especially strong for VIX levels less than the mean.

These results suggest that ATM put options are most responsible for driving changes in VIX, and this appears particularly true when VIX is below its historical mean. Given these findings, and because we are interested in investing in a portfolio that has returns corresponding to the VIX index, in order to hedge against downturns in the market, holding a portfolio that is long in ATM S&P 500 puts appears to be a natural fit. However, it has been well documented that puts are expensive.[10] Thus, it is necessary to not only go long puts, but sell either OTM puts or corresponding calls to offset the cost. Given the

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[9] A one dollar change times a 0.01 change in moneyness for a one-month option, multiplied by the 48.47 coefficient on *I*, results in a 0.02% change in VIX.

[10] For example, Jackwerth (2000), Aït-Sahalia, Wang and Yared (2001), Coval and Shumway (2001), Bakshi and Kapadia (2003), Bondarenko (2003), Bollen and Whaley (2004), and Liu, Pan and Wang (2005) generally find that the historical costs of puts, particularly OTM and ATM puts, are too expensive to be justified.

results in Table 5, we opt to form a zero-cost portfolio that will buy ATM puts and OTM calls and sell OTM puts and ATM calls, where the ratio of calls to puts will be determined by the level of VIX and the coefficients on the price.

Since one of the hurdles in replicating VIX is the difficulty in daily rebalancing costs of the options, our replicating portfolio will enter into the zero-cost investment at the beginning of the month, using both the near-term and 2nd near-term options only. The investment in the options will only occur if the level of the VIX index is below 35 since our earlier results have shown the negative correlation and asymmetric response between the VIX index and the S&P 500. The portfolio will hold the equivalent number of options corresponding to 100 shares in the index, while accounting for the margin required by selling options. As such, less total investment can be made in the S&P 500 since cash needs to be set aside to maintain the margin account. In addition, there may be instances when additional cash is needed to cover potential margin calls. Should this occur, shares from the underlying will be sold to make up cash shortfalls.

B. Performance of VIX Replication

Figure 4 shows the return to a portfolio that initially invests $100,000 in either the S&P 500 or the S&P 500 and the synthetic VIX portfolio, starting January 1996 and ending October 2009. The return to the portfolio that incorporates the replicating VIX strategy outperforms the buy-and-hold S&P 500 portfolio significantly. The annualized return to hedged portfolio is 12.4% with an annual volatility is 18.6%. The return is 6.5% higher than the S&P 500 buy-and-hold portfolio with almost identical risk. The alpha from a market model regression is 6.9% and is highly significant, but becomes insignificant if VIX is included in the specification. This suggests that our replicating portfolio does a good job of capturing the hedging benefits of holding the VIX index.

Using different levels of VIX as the threshold level has a minimal effect on the overall return to the replicating portfolio unless the threshold level of VIX is above 45 or below 20. If a threshold level of 20 is used, then the hedge does not cover large declines in the S&P 500. When an investment is made with VIX above 45, the return to the hedge portfolio is negligible because these periods tend to be uncorrelated to the increases in the S&P 500.

There is a specific downside to the synthetic VIX portfolio approach. It cannot capture the positive skewness in the VIX index. Hedging skewness in VIX, which corresponds to large increases in the index or sharp declines in the S&P 500, is very costly and thus not worth implementing. For example, from September through November of 2008, when the S&P 500 fell over 400 points and VIX increased from a level of 20 to the high 80's, the replicating strategy would have lost significantly because the OTM puts would have paid off significantly more than the ATM options. This also would have occurred in March and September/October of 2001 and July 2002, when VIX spiked and the S&P fell sharply. In total, the hedge portfolio would lose 7 out of 168 months in our sample; however, while the frequency is rare, the losses would have been large.

Such risks, if hedged, would cost more than the total gains, and are risks we feel are acceptable. Since the replicating portfolio captures the asymmetric relationship between volatility and returns, it captures most of the downside risk investors wish to hedge. The fact that the portfolio does not capture the skewness is unfortunate, but is a risk that as of now appears unavoidable. A potential alternative to hedging skewness is reverting to VIX calls. Figure 3 highlights the ability of the VIX calls to hedge the market decline in late 2008 as a result of the positive skewness in the VIX index. Thus a portfolio that combined our replicating strategy with some VIX calls may be the ideal hedging combination.

IV. Conclusion

Our results indicate the effectiveness and importance of holding volatility as an asset class in avoiding market shortfalls by hedging downside risk. Since the current asymmetric relationship between the VIX index and the S&P 500 generates the strongest correlations when the market is falling, holding the VIX long is a natural candidate for hedging market risk while still providing upside return potential. Our results show that if VIX were investable, a portfolio comprised of VIX and the S&P 500 would provide returns and risk that far outpace the traditional buy-and-hold portfolio.

Of course, those initial results are hypothetical, and moreso, un-replicable via VIX futures or options because they do not capture the same properties that characterize the index, particularly, mean reversion. The portfolio of holding VIX futures in combination with either the S&P 500 or futures on the S&P 500 significantly underperforms. This result is disappointing since VIX futures do not offer the payoff to volatility that investors would expect. It is not clear if this is due to low liquidity or mispricing of futures contracts, since it is not possible to arbitrage mispricing by holding the underlying.

By dissecting the VIX into the individual options that are responsible for the index change, we demonstrate that S&P 500 puts closest to ATM are most responsible for the VIX movements, especially when volatility increases. With this understanding, we are able to form a portfolio which holds the S&P 500 option contracts that best capture increases in VIX while selling other contracts to form a zero-cost investment portfolio. Our synthetic

VIX portfolio is quite liquid, unlike VIX calls to date, and performs extremely well, capturing the increases in VIX without proportionally penalizing the portfolio when the market increases. While this portfolio does require the use of margin, it neutralizes the downside market risk investors wish to remove while allowing for the upside gain. The aspect it cannot capture is positive skewness in volatility. However, through holding VIX calls, we demonstrate new hedges for traditional passive long strategies which are a much more effective (or cost-effective) portfolio insurance than buying S&P 500 puts.

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Table 1: Correlations

Table 1, below, demonstrates the correlation of monthly S&P 500 returns with the returns that would be realized from simultaneous hypothetical investment in the VIX index. Panel A presents results for the full sample. Panel B presents results for those months that experience an increase in the S&P 500 level while Panel C presents results for months that experience a decrease in the S&P 500 level.

Panel A: Full sample

	N	Mean Return	Std Dev	Median	Correlation
S&P 500	235	0.50%	4.54%	0.90%	
VIX Index	235	1.64%	17.79%	-0.24%	-0.616

Panel B: Months with positive S&P 500 returns

	N	Mean Return	Std Dev	Median	Correlation
S&P 500	139	3.34%	2.80%	2.64%	
VIX Index	139	-5.96%	12.50%	-6.16%	-0.274

Panel C: Months with negative S&P 500 returns

	N	Mean Return	Std Dev	Median	Correlation
S&P 500	96	-3.51%	3.38%	-2.50%	
VIX Index	96	12.65%	18.60%	10.26%	-0.574

Table 2: VIX Index Portfolio Returns

Table 2, below, provides return and abnormal return measures for hypothetical holdings of the VIX index. Panel A gives monthly results for the VIX index when all observation months are included, and then, separately, for months when positive (negative) S&P 500 returns are realized. Panel B gives monthly results for the naïve strategies which invest equally in the hypothetical VIX and the S&P 500 when VIX levels are below the numerical level presented and entirely in the S&P 500 otherwise. Mean monthly returns are presented, as are the CAPM alpha and beta which result from the regression of monthly portfolio returns on the value-weighted monthly market return, and the Carhart (1997) 4-factor alpha and market beta which result from the regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) and Carhart's (1997) momentum factor (UMD), which are obtained from Ken French's website. Test statistics are given in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: VIX index portfolio results	Mean	CAPM MKT Beta	CAPM Alpha	4-Factor MKT Beta	4-Factor Alpha
Full sample	1.64	-2.13***	2.15**	-2.13***	2.29**
	(1.41)	(-9.65)	(2.28)	(-8.55)	(2.25)
Months S&P 500 returns>0	-5.96***	-0.77*	-3.95**	-0.36	-5.07***
	(-5.63)	(-1.82)	(-2.36)	(-0.75)	(-2.83)
Months S&P 500 returns<0	12.65***	-1.70***	6.44**	-1.60***	7.78***
	(6.66)	(-3.64)	(2.67)	(-3.13)	(3.09)

Panel B: Portfolio Strategy	Mean	CAPM MKT Beta	CAPM Alpha	4-Factor MKT Beta	4-Factor Alpha
Naïve 20	1.54***	0.48***	1.00**	0.41***	1.25***
	(3.39)	(5.00)	(2.31)	(3.80)	(2.82)
Naïve 25	1.57***	0.04	1.21**	-0.40	1.44***
	(3.13)	(0.37)	(2.38)	(-0.29)	(2.81)
Naïve 30	1.46	-0.23**	1.22**	-0.23*	1.35**
	(2.85)	(-1.97)	(2.38)	(-1.77)	(2.58)
Naïve 35	1.13	-0.31**	0.92*	-0.28**	0.96*
	(2.18)	(-2.72)	(1.79)	(2.20)	(1.82)
Naïve 40	1.29	-0.43***	1.13**	-0.44***	1.25**
	(2.51)	(-3.85)	(2.24)	(-3.53)	(2.43)

Table 3: VIX Index Buy and Hold Returns

Table 3, below, gives the overall and monthly mean returns for investment strategies which include holdings in either the S&P 500, the hypothetical VIX asset, or both in the naïve strategy cases. The naïve strategies invest equally in the hypothetical VIX and the S&P 500 when VIX levels are below the numerical level presented and entirely in the S&P 500 otherwise.

	Overall BHR (%)	Mean Monthly BHR (%)	Std Dev of Monthly BHR(%)
S&P 500	178.75	0.44	4.69
VIX	48.26	0.17	18.56
Naïve 20	2022.19	1.31	7.02
Naïve 25	1933.85	1.29	7.74
Naïve 30	1428.65	1.17	7.91
Naïve 35	595.96	0.83	8.00
Naïve 40	922.84	0.99	7.89

Table 4: VIX Futures Portfolio Returns

Table 4, below, provides monthly return and abnormal return measures for holdings of VIX futures. Panel A gives monthly results for VIX futures when all observation months are included, and then, separately, for months when positive (negative) S&P 500 returns are realized. Panel B gives monthly results for the naïve strategies which invest equally in VIX futures and the S&P 500 when VIX levels are below the numerical level presented and entirely in the S&P 500 otherwise. Mean monthly returns are presented, as are the CAPM alpha and beta which result from the regression of monthly portfolio returns on the value-weighted monthly market return, and the Carhart (1997) 4-factor alpha and market beta which result from the regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) and Carhart's (1997) momentum factor (UMD), which are all obtained from Ken French's website. The mean return from the VIX index portfolio over the same time period are also presented in Panel A for comparison purposes. Test statistics are given in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Panel A: VIX Futures portfolio results	VIX Futures Mean Return	VIX Index Mean Return	CAPM MKT Beta	CAPM Alpha	4-Factor MKT Beta	4-Factor Alpha
Full sample	-3.12	2.83	-3.80***	-3.65**	-4.21***	-3.80**
(N=59)	(-0.95)	(0.94)	(-11.08)	(-1.97)	(-10.19)	(-2.12)
Months S&P 500 Futures returns>0	-14.07***	-7.23***	-0.96*	-11.53***	-1.13	-10.83***
(N=34)	(-8.44)	(-3.14)	(-1.70)	(-5.01)	(-1.33)	(-3.77)
Months S&P 500 Futures returns<0	11.78*	16.51***	-5.05***	-9.43**	-5.30***	-10.29**
(N=25)	(1.87)	(3.09)	(-8.92)	(-2.45)	(-7.61)	(-2.69)

Panel B: Portfolio strategy results	VIX Futures Mean Return	CAPM MKT Beta	CAPM Alpha	4-Factor MKT Beta	4-Factor Alpha
Naïve Futures 20	-2.30**	0.40**	-2.51***	0.29	-2.44***
	(-2.72)	(2.64)	(-3.12)	(1.58)	(-3.14)
Naïve Futures 25	-1.97**	0.06	-2.20**	-0.17	-2.21**
	(-2.18)	(0.36)	(-2.42)	(-0.79)	(-2.46)
Naïve Futures 30	-1.02	-1.08***	-1.31	-1.57***	-1.42
	(-0.79)	(-5.30)	(-1.22)	(-7.19)	(-1.53)
Naïve Futures 35	-1.27	-1.09***	-1.56	-1.60***	-1.67*
	(-0.98)	(-5.33)	(-1.45)	(-7.37)	(-1.81)
Naïve Futures 40	-1.57	-1.17***	-1.87*	-1.59***	-2.02**
	(-1.22)	(-5.98)	(-1.81)	(-7.29)	(-2.18)

Table 5: VIX Factor Regression

Table 5, below, shows the results from the following regressions:

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \Delta P_{\kappa,\phi,\tau,t} * CD_t + \Delta KS_{\kappa,\phi,\tau,t} * CD_t + \varepsilon_t$$

and

$$\ln\left(\frac{VIX_t}{VIX_{t-1}}\right) = \alpha_t + \Delta P_{\kappa,\phi,\tau,t} + \Delta KS_{\kappa,\phi,\tau,t} + DM_{\kappa,\phi,\tau,t} + I_{\kappa,\phi,\tau,t} + \varepsilon_t$$

Where $\Delta P_{\kappa,\phi,\tau,t}$is the change in price of ϕ(call/put) option with a strike of K and maturity τ at time t. ΔKS represents the change is moneyness of the option, where put moneyness is equal to S/K and call moneyeness is equal to K/S. Thus, more positive changes bring both definitions of moneyness closer to ATM. DM is the time to maturity of the option, and I is the interaction of all three variables. CD is a dummy variable equal to one if the option is a call and zero if the option is a put. The first regression interacts the dummy variables with change in price and moneyness variables to capture differences between call and put options. The second regression examines the effect of price and moneyness changes of calls and puts separately on VIX changes. Each specification clusters the standard errors by date to avoid overstating the t-statistics. Test statistics are given in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

	ΔVIX	ΔVIX >0	ΔVIX <0	Puts Only VIX>22 ΔVIX	Puts Only VIX<22 ΔVIX	Calls Only VIX>22 ΔVIX	Calls Only VIX<22 ΔVIX
ΔP	0.248	0.339	0.198	0.203	0.818	0.112	0.346
	(7.36)**	(4.91)**	(7.98)**	(6.67)**	(9.18)**	(3.81)**	(4.70)**
ΔKS	331.813	210.608	172.541	301.053	448.465	-364.368	-636.329
	(32.68)**	(13.11)**	(16.07)**	(27.59)**	(21.61)**	(33.74)**	(21.62)**
DM	-1.031	-1.292	0.55	-0.922	-4.73	-0.197	-3.983
	(1.35)	(1.25)	(0.76)	(0.91)	(3.12)**	(0.17)	(2.62)**
ΔP*CD	-0.14	-0.262	-0.043				
	(3.88)**	(2.80)**	(1.06)				
ΔKS*CD	-732.004	-510.739	-407.071				
	(38.11)**	(17.27)**	(19.58)**				
I	48.473	15.45	19.685	38.079	456.779	44.168	413.826
	(4.76)**	-0.69	(4.80)**	(4.61)**	(4.42)**	(2.64)**	(3.38)**
Constant	0.26	2.958	-2.622	0.427	0.533	0.306	0.381
	(2.85)**	(24.20)**	(31.39)**	(3.63)**	(3.94)**	(2.33)*	(2.37)*
OBS	33681	15813	17718	9437	8177	9358	6682
R-squared	0.57	0.39	0.35	0.61	0.55	0.65	0.57

Figure 1: VIX Index Portfolio Returns

Figure 1, below, tracks the performance of investment strategies incorporate the holding of the hypothetical VIX asset. Naïve 20 (40) holds hedging positions in the hypothetical VIX asset when a calendar month begins with a VIX level less than 20 (40) and holds only the S&P 500 otherwise. Hedging consists of utilizing half of available funds to go long in the hypothetical VIX asset. Buy-and-hold positions with monthly formation are tracked beginning with the availability of historical VIX data.

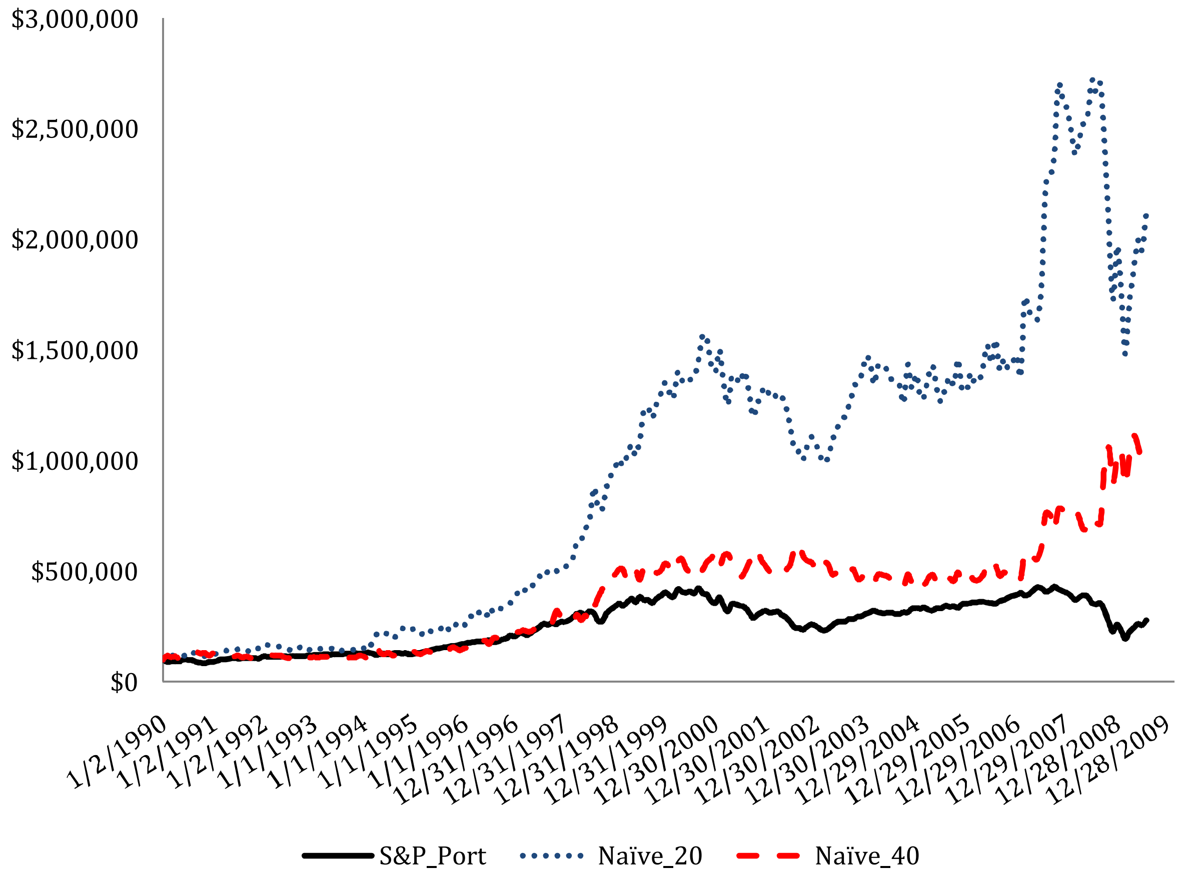

Figure 2: VIX Futures Portfolio Returns

Figure 2, below, demonstrates the returns to an investment strategy which commences investment in S&P 500 futures in March of 2004 when VIX futures were introduced. An approach which invests in S&P 500 futures and reinvests after each futures contracts' expiration is shown. Additionally, a Naïve 20 (40) strategy is considered which invests half of the available funds in VIX futures when the VIX level is below 20 (40) at the previous futures expiration date.

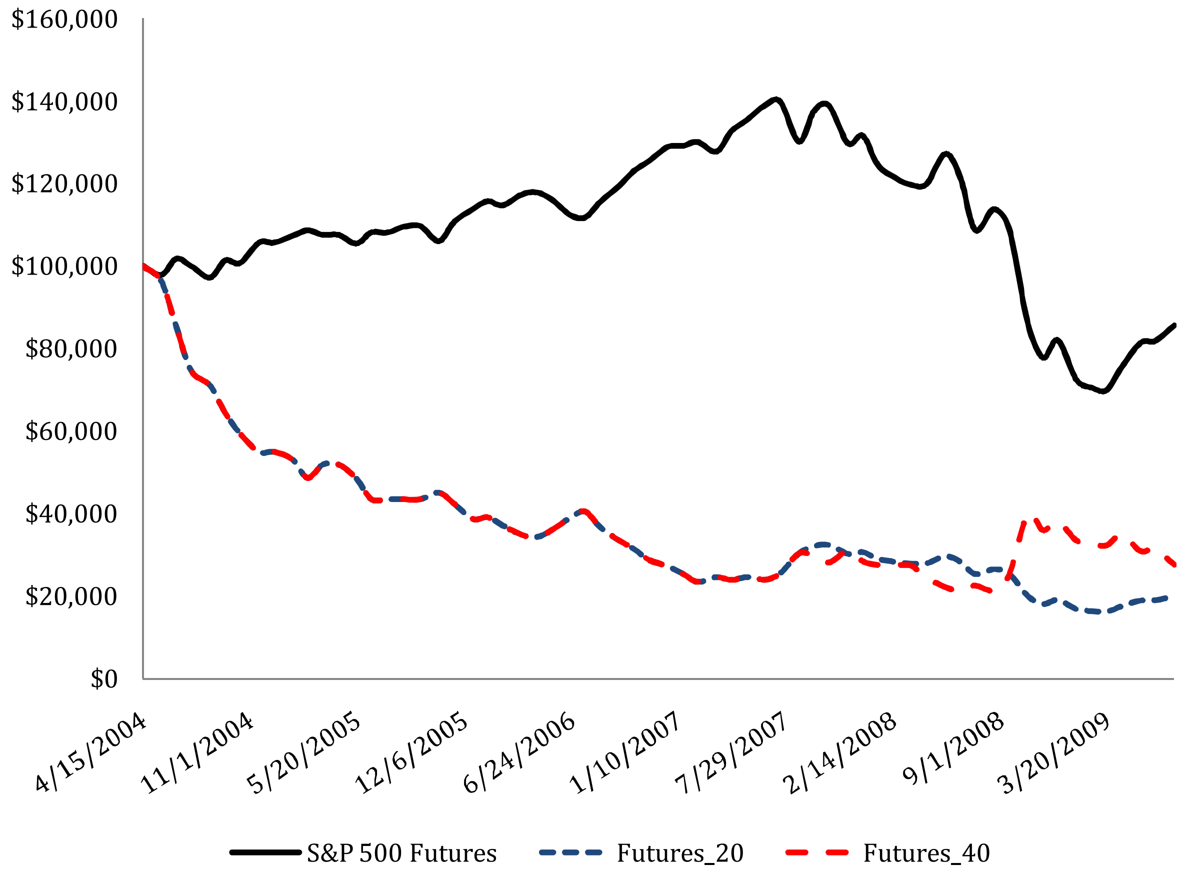

Figure 3: Hedging Using VIX Calls

Figure 3, below, demonstrates the returns to an investment strategy which commences investment in the S&P 500-tracking ETF, SPY beginning in May of 2006 with the introduction of VIX options data. Approaches which invest in SPY at the beginning of the study period and concurrently invest in options to form delta-neutral portfolios are analyzed. The use of VIX calls and, S&P 500 puts as hedging instruments are alternatively considered.

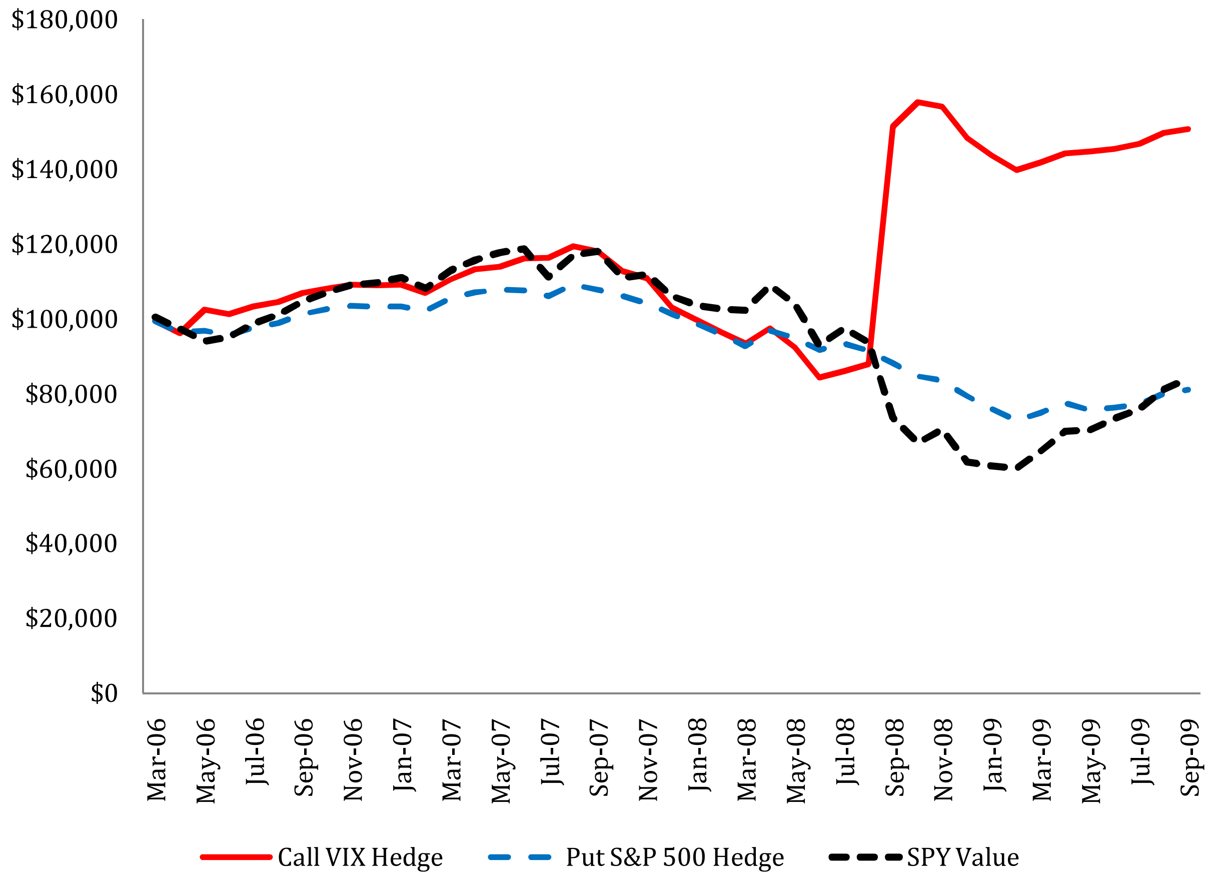

Figure 4: Synthetic VIX Portfolio Returns

Figure 4, below, tracks the value of portfolios that consist of the S&P 500, holdings in the hypothetical VIX asset, and the S&P 500 hedged with holdings of the synthetic VIX-replicating strategy at a Naïve 35, respectively. The VIX-replicating strategy is comprised of positions in S&P 500 index options. After the expiration of one group of index options used to construct the synthetic VIX position, the VIX level is evaluated and if the level is below 35 then the hedge synthetic VIX-replicating strategy is employed. Analysis of portfolio performance begins January 1996 and ends October 2009.